Fast, safe and precise landing of a quadrotor on an oscillating platform

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***Abstract*— In this paper, we propose a novel control structure that can achieve fast, safe and precise landing of a VTOL (vertical takeoff and landing) UAV onto a vertically oscillating landing pad. The control structure consists of three modules to achieve these goals: a motion estimation module, a trajectory generation module and a tracking control module. In the tracking control module, an ARC (Adaptive Robust Controller) is designed to robustly adapt the nonlinear ground effect to enable a quadrotor accurately track a given reference trajectory. In the trajectory generation module, a time-optimal reference trajectory for the quadrotor is generated such that it converges from the initial height precisely to the platform height with zero relative velocity (for smooth landing). The landing time duration is as short as possible, and physical safety constraints (position, velocity, acceleration bounds etc.) are satisfied during the entire landing process. The above two modules use the motion information of the quadrotor and the platform in absolute coordinate system (inertial frame). In the motion estimation module, we estimate the UAV and platform positions online from only the measurement of the relative distance between the UAV and the platform, as well as the inertia measurement of the UAV. An UKF (unscented Kalman Filter) is constructed and the estimated parameters are fed to the other two modules in real time. Comparative simulation and experimental results are presented to validate the performances of the proposed control structure.**

I. INTRODUCTION

Nowadays, VTOL UAVs are playing more and more important roles in many aspects. One critical capability is the autonomous landing capability onto a moving target such as a ship deck under high sea conditions. This ability can have a significant positive impact on maritime applications for UAVs. It is desirable for the shipboard landing process to be fast, safe and precise. However, shipboard landing is difficult because of the nonlinear dynamics of the UAV, uncertainties in the system and the time varying nature of the shipboard motion. To address the VTOL UAV shipboard landing problem, a variety of feedback controllers have been developed. Herisse *et al*. [1] developed a feedback control algorithm based on optical flow. Oh *et al*. [2] developed a controller that guided the autonomous landing operation of a helicopter using a tether. Lee *et al*. [3] and Ling *et al* [4] developed a controller that enabled a quadrotor landing on a moving platform based on the vision clue. Although those approaches achieve closed-loop stability in landing process and possess certain robustness, there is no guarantee in fulfilling the fast landing requirement under varieties of physical constraints such as the input saturation and position and velocity limitations.

Instead of designing only a feedback controller with no knowledge of the platform motion, an alternate approach towards autonomous landing focuses on identification of the pattern of platform motion and feedforward compensation based on this motion profile to achieve good landing performance. When the landing maneuver is performed on a moving deck on open ocean, the vertical movement of the wave can be modeled as a superposition of several sine functions with different amplitudes, frequencies and phases [5], [6]. Methods such as recursive least square [7], Kalman filter [8], extended Kalman Filter [9] [10], Unscented Kalman Filter [5] and adaptive identifier [11] can be used for wave motion estimation. If the estimation of the sea wave motion is available, the autonomous landing problem may be reduced to a tracking control problem and conventional methods used for trajectory tracking in motion control can be used. For fast tracking and short landing time, optimal control based approaches have been developed such as the time optimal trajectory tracking control [12] [13], minimal jerk trajectory generation and control [14], minimal snap trajectory generation and control [15], etc. Although following a time optimal trajectory can minimize the landing time, it operates in open loop and may introduce non-zero tracking errors when there are uncertainties in system dynamics, as evidenced in [13].

In summary, neither pure feedback control nor time-optimal control alone can satisfy the speed, safety and precision requirements simultaneously. In this paper, we propose a control structure inspired by [16] that combines the advantages of feedback control and time-optimal control together to achieve fast, safe and precise landing on a moving platform. The detail design will be introduced in the following sections.

There are two novelties in the proposed control structure. First, the proposed approach enables fast, safe and precise landing simultaneously on an moving platform with unknown (but parameterizable) motion. Secondly, considering the fact the absolute motion of the aircraft and the platform is not usually available in outdoor environment, especially under GPS denied situations, the proposed approach requires only the relative distance between the UAV and the platform and the accelerometer reading of the UAV as measurements. With these measurements, we are able to estimate the absolute motion of the UAV (height and velocity) as well as the platform (height, velocity and acceleration) in the inertial frame. Such an estimation strategy was also not done before.

This paper is organized as following. Section II formulates

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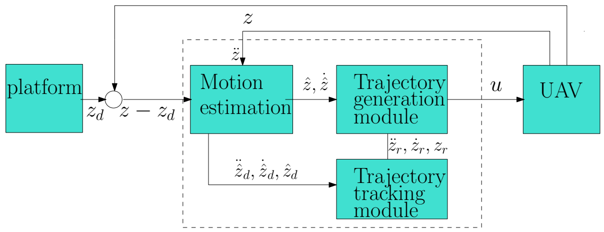

Fig. 1. The control structure consists of three modules in the dashed box.The motion estimation module generates the estimation of the platform motion $\hat{z}_d(t), \dot{\hat{z}}_d(t), \ddot{\hat{z}}_d(t)$ for the trajectory generation module and estimation of the UAV motion $\hat{z}(t), \dot{\hat{z}}(t)$ for tracking control module. The trajectory generation module outputs the reference trajectory $z_r(t), \dot{z}_r(t), \ddot{z}_r(t)$ for the tracking control module. The tracking control module outputs the thrust command u

the problem and explains the control objective. Section III, section IV, section V presents the design of the tracking control module, trajectory generation module and motion estimation module. Section VI gives the experimental results. Section VII concludes the paper.

II. PROBLEM FORMULATION

For this paper, we focus on the VTOL UAV vertical landing problem. A quadrotor is used for control algorithm development, implementation, and validation. The detailed analysis of quadrotor dynamics can be found in [17]. This section will first briefly introduce the system dynamics in vertical direction and then explains the control objective.

The dynamics of the system in the vertical direction can be simplified as (1).

$$\begin{aligned} \dot{z} &= v \\ \dot{v} &= \frac{S(u)}{m} - g \end{aligned} \tag{1}$$

z is the vertical height and v is the vertical velocity. u is the thrust command input to the quadrotor and $S(u)$ is the thrust that a quadrotor generated. When the quadrotor is approaching to the ground, the rotor will generate more thrust for a given power. This effect is called the ground effect [18]. The ground effect can be modeled as a function of height above the platform $z - z_d$ as (2) where R is a constant [18].

$$S(u) = k_G u = \frac{1}{1 - (\frac{R}{4(z - z_d)})^2} u \tag{2}$$

The platform motion z_d and velocity $\dot{z}_d$ is modeled as summation of multiple sinusoidal functions as following:

$$\begin{aligned} z_d(t) &= \sum_{i=1}^{n} A_i \sin(\omega_i t + \phi_i) \\ \dot{z}_d(t) &= \sum_{i=1}^{n} A_i \omega_i \cos(\omega_i t + \phi_i) \end{aligned} \tag{3}$$

where A_i represents the amplitude of the $i-th$ component of the sinusoidal motion, ω_i is the frequency and ϕ_i is the phase, similar to [5].

Now we present the control objective. The three requirements namely, speed, safety and preciseness of the landing, can be formulated analogously to requirements for motion control design. 'Precise' means that states of the quadrotor $(z(t), \dot{z}(t))$ converges to those of the platform $(z_d(t), \dot{z}_d(t))$ with sufficient accuracy for some t. In other words, the final value of $(z(t), \dot{z}(t))$ should be within a small neighborhood of $(z_d(t), \dot{z}_d(t))$. 'Fast' means that the time, t_f needed for $(z(t), \dot{z}(t))$ to reach the small neighborhood of $(z_d(t), \dot{z}_d(t))$ is as short as possible. 'Safe' means that the physical safety requirements such as the position constraint (the quadrotor does not hit the platform), velocity limitation (such as maximum descending velocity) and and the acceleration limitation are satisfied.

In brief, the control objective is to design a control input u such that based on $z(t) - z_d(t)$and $\ddot{z}(t)$ as measurements, $z \to z_d$ and $\dot{z} \to \dot{z}_d$ as fast and accurately as possible, while the motion constraints (such as the position constraint and velocity/acceleration limitations) are satisfied.

To meet the three design targets, we propose a novel control structure consisting of three different modules [16] as shown in Fig. 1. The motion estimation module estimates the absolute platform and UAV motion based on UKF with the relative distance and the acceleration of the UAV as measurements. The reference trajectory generation module generates a time-optimal reference trajectory with safety constraints satisfaction based on the online estimation of the platform motion. The tracking control module utilizes ARC (adaptive robust controller) to let the UAV motion track the reference trajectory in the presence of strong uncertainties such as the ground effect. The detail design of three modules will be introduced as following.

III. TRACKING CONTROL MODULE DESIGN

The ground effect coefficient k_G in (2) significantly alters the thrust on the quadrotor, thereby affecting system dynamics that must be considered in the controller design.

Online Adaptation of Ground Effect: For the control design, offline identification may take a long time and many experiments. So to deal with uncertainties in a real system, we estimate k_G online. Specifically, dividing both sides of (1) by $\frac{k_G}{m}$:

$$\begin{aligned} \dot{z} &= v \\ \frac{m}{k_G}\dot{v} &= u - \frac{m}{k_G} g. \end{aligned} \tag{4}$$

It is seen that left-hand side of (4) has an uncertain equivalent inertia $\frac{m}{k_G}$, defined as θ, which may be estimated online to deal with the changes and uncertainties.

The inputs to this module as shown in Fig.1 is the reference position, velocity and acceleration $[z_r(t) \quad \dot{z}_r(t) \quad \ddot{z}_r(t)]^T$, as well as the estimated quadrotor absolute position z and velocity $\dot{z}$. In order to achieve accurate tracking with respect to the reference trajectory as well as robustness, we use an ARC (Adaptive Robust Control) method similar to [19], [20] to generate the control law u.

Let $w_1 = z - z_r$ denote the tracking error with respect to the reference trajectory. Define a switching-function-like quantity $w_2 = \dot{w}_1 + k_1 w_1 = v - \dot{z}_r + k_1 w_1$, where k_1 is a positive gain. The tracking error dynamics can be written as (5).

$$\begin{aligned} \dot{w}_1 &= -k_1 w_1 + w_2 \\ \theta \dot{w}_2 &= u + \varphi\theta, \end{aligned} \tag{5}$$

where $\theta = \frac{m}{k_G}$ and $\varphi = -(g + \ddot{z}_r - k_1 \dot{w}_1)$. If w_2 is small or converges to zero, then the output tracking error w_1 will also be small or converge to zero since $G(s) = w_1(s)/w_2(s) = 1/(s+k_1)$ is a stable transfer function. We proposed the following ARC control law to make w_2 as small as possible:

$$u = u_a + u_s, \tag{6}$$

where u_a is the adjustable model compensation needed for perfect tracking and u_s is the robust control law to be synthesized later. It is noted that the actual tracking error variables w_1 and w_2 are not directly available since we only have the estimates of z and $\dot{z}$. From the estimates, we construct the estimated w_1 and w_2 to be used in synthesizing u_a and u_s:

$$\begin{array}{rcl} \hat{w}_1 & = & \hat{z} - z_r, \\ \dot{\hat{w}}_1 & = & \dot{\hat{z}} - \dot{z}_r, \\ \hat{w}_2 & = & \dot{\hat{w}}_1 + k_1 \hat{w}_1. \end{array} \tag{7}$$

The model compensation term u_a is designed as follows:

$$u_a = -\hat{\varphi}\hat{\theta}, \tag{8}$$

where $\hat{\varphi} = -(g + \ddot{z}_r - k_1 \dot{\hat{w}}_1)$. The parameter estimation $\hat{\theta}$ is obtained from the following discontinuous-projection-type adaptation law:

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\gamma\hat{\varphi}\hat{w}_2) = \begin{cases} 0, \text{ if } \hat{\theta} = \theta_{max} \text{ and } \gamma\hat{\varphi}\hat{w}_2 > 0 \\ 0, \text{ if } \hat{\theta} = \theta_{min} \text{ and } \gamma\hat{\varphi}\hat{w}_2 < 0 \\ \gamma\hat{\varphi}\hat{w}_2, \text{ else} \end{cases}, \tag{9}$$

where $\gamma > 0$ is the adaptation gain, and $\theta_{max} = m, \theta_{min} = \frac{m}{k_{Gmax}}$ are known.

The robust control function u_s has the following structure:

$$u_s = u_{s1} + u_{s2}, \; u_{s1} = -k_2 \hat{w}_2 \tag{10}$$

u_{s1} is a simple proportional feedback to stabilize the nominal system and u_{s2} is a robust performance feedback term having the following properties:

$$\begin{array}{r} \hat{w}_2\{u_{s2} - \hat{\phi}\tilde{\theta}\} \leq \varepsilon \\ \hat{w}_2 u_{s2} \leq 0 \end{array} \tag{11}$$

where ε is a design parameter that can be arbitrarily small. With the proposed control law , we have the following theorem that can be proved using the same technique as in [19].

Theorem 1: Given the adaptation law as defined in (9), the controller in (10) guarantees the following.

A) In general, if the state estimates $\hat{z}$ and $\dot{\hat{z}}$ are bounded, then all the signals are bounded. Furthermore, the positive definite function V_s defined by

$$V_s = \frac{1}{2} m \hat{w}_2^2 \tag{12}$$

is bounded above by

$$V_s(t) \leq \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)] \tag{13}$$

where $\lambda = \frac{2k_2}{\theta_1 \max}$.

B) If the state estimation is perfect, i.e., $\hat{z} = z$ and $\dot{\hat{z}} = \dot{z}$, then in addition to results in A), zero final tracking error is also achieved, i.e., $w_1 \to 0$ and $w_2 \to 0$ as $t \to \infty$.

IV. Trajectory generation module design

In this module, a time optimal reference trajectory $z_r(t)$ is generated while guaranteeing that motion constraints are met. It is key to note that the trajectory generation module is inactive when the quadrotor is hovering. After the landing maneuver is initiated, this module is triggered. This starting time is denoted as t_0. The trajectory $z_r(t)$ generated should satisfy the following criteria [16]:

1. The initial values $z_r(t_0)$ and $\dot{z}_r(t_0)$ are the same as the state estimates $\hat{z}_r(t_0)$ and $\dot{\hat{z}}_r(t_0)$ such that the tracking error estimation variables $\hat{w}_1$ and $\hat{w}_2$ fed into the tracking control module are zero to avoid big input (input saturation) at the beginning. This is referred to as 'initial condition matching' as described in [16].
2. The final values $z_r(t_f)$ and $\dot{z}_r(t_f)$ are the same as $\hat{z}_d(t_f)$ and $\dot{\hat{z}}_d(t_f)$ such that the quadrotor completes landing on the platform exactly (assuming the platform motion estimation is sufficiently accurate).
3. The reference position, velocity and acceleration are constrained, i.e., $z_r(t) \geq \hat{z}_d(t)$, $|\dot{z}_r(t)| \leq \dot{z}_{rmax}$, $\ddot{z}_{rmin} \leq \ddot{z}_r(t) \leq \ddot{z}_{rmax}$, for all $t \in [t_0, t_f]$. In the above, the position constraint means the quadrotor is always above the platform.
4. The landing is as fast as possible, i.e., t_f is as short as possible.

Let $z_a = z_r - \hat{z}_d, \dot{z}_a = \dot{z}_r - \dot{\hat{z}}_d$ denote the relative 'trajectory modification' with respect to the platform motion. The problem then is reduced to generation of $\ddot{z}_a$ to satisfy the above criteria and then integrate to get z_a, $\dot{z}_a$ and subsequently z_r and $\dot{z}_r$. According to the above criteria, it is clear that the following optimization algorithm needs to be solved in real time:

$$\begin{array}{c} \min\limits_{\ddot{z}'_a(\tau), \tau \in (t, t_f)} (t_f) \\ \text{subject to} \\ z'_a(\tau) = z_a(t), \dot{z}'_a(\tau) = \dot{z}_a(t) \\ z'_a(t_f) = 0, \dot{z}'_a(t_f) = 0, \\ z'_a(\tau) \geq 0, \forall \tau \in [t, t_f], \\ -\dot{z}_{rmax} - \dot{\hat{z}}_d(\tau) \leq \dot{z}'_a(\tau) \leq \dot{z}_{rmax} - \dot{\hat{z}}_d(\tau), \forall \tau \in [t, t_f], \\ \ddot{z}_{rmin} - \ddot{\hat{z}}_d(\tau) \leq \ddot{z}'_a(\tau) \leq \ddot{z}_{rmax} - \ddot{\hat{z}}_d(\tau), \forall \tau \in [t, t_f], \end{array} \tag{14}$$

where the variables with a prime sign are the optimization variables. When $t = t_0$, the initial values $z_a(t_0)$ and $\dot{z}_a(t_0)$ are set as $\hat{z}(t_0) - \hat{z}_d(t_0)$ and $\dot{\hat{z}}(t_0) - \dot{\hat{z}}_d(t_0)$. After solving the problem, the optimal solution is used as $\ddot{z}_a$.

It is noted that in optimization problem formulation, the constraints depend on the values of $\hat{z}_d$, $\dot{\hat{z}}_d$ and $\ddot{\hat{z}}_d$ in the future ($\tau > t$). The actual platform $z_d(t)$ is assumed to be a sinusoidal function of t. If the estimations of the parameters of this sinusoidal function (such as the frequency, phase and amplitude) keep changing, then the future values of $\hat{z}_d(t)$ and $\dot{\hat{z}}_d(t)$ are not immediately available at time t. To bypass this problem, we use the current parameter estimations of the sinusoidal platform motion to construct the future values of $\hat{z}_d$, $\dot{\hat{z}}_d$ and $\ddot{\hat{z}}_d$.

According to the Pontryagin's theorem, the optimal solution $\ddot{z}_a(t)$ is a bang-bang type control law [16].

$$\ddot{z}_a(t) = \begin{cases} \ddot{z}_{rmin} - \ddot{\hat{z}}_d(t) & \forall (z_a(t), \dot{z}_a(t)) \in \Omega_1(t) \\ -\ddot{\hat{z}}_d(t) & \forall (z_a(t), \dot{z}_a(t)) \in \Omega_2(t) \\ \ddot{z}_{rmax} - \ddot{\hat{z}}_d(t) & \forall (z_a(t), \dot{z}_a(t)) \in \Omega_3(t) \end{cases}, \quad (15)$$

where $\Omega_1(t)$ is the region of maximum downward acceleration, $\Omega_2(t)$ is the region of zero acceleration to maintain maximum downward velocity, $\Omega_3(t)$ is the region of maximum downward deceleration in order to make a stop. The following procedure [21] determines which region (Ω_1, Ω_2 or Ω_3) the current state $(z_a(t), \dot{z}_a(t))$ belongs to:

If $z_a(t) = 0$, it means that the quadrotor is already on the platform so that the landing process is terminated. Else, we define a future 'test trajectory' $(z_a(\tau), \dot{z}_a(\tau))$, $\forall \tau \geq t$ starting from the current state $(z_a(t), \dot{z}_a(t))$ with the minimum downward deceleration:

$$\begin{aligned} z_a(\tau) &= z_a(t) + \dot{z}_a(t)(\tau - t) + \int_t^\tau \int_t^{\tau_1} (\ddot{z}_{rmax} - \ddot{\hat{z}}_d(\tau_2)) d\tau_2 d\tau_1 \\ &= z_a(t) + \dot{z}_a(t)(\tau - t) + \tfrac{1}{2}\ddot{z}_{rmax}(\tau - t)^2 - \hat{z}_d(\tau) + \hat{z}_d(t) \\ &\quad + \dot{\hat{z}}_d(t)(\tau - t) \\ \dot{z}_a(\tau) &= \dot{z}_a(t) + \int_t^{\tau_1} (\ddot{z}_{rmax} - \ddot{\hat{z}}_d(\tau_1)) d\tau_1 \\ &= \dot{z}_a(t) + \ddot{z}_{rmax}(\tau - t) - \dot{\hat{z}}_d(\tau) + \dot{\hat{z}}_d(t) \end{aligned} \quad (16)$$

It can be checked that if $(\ddot{z}_{rmax} - \ddot{\hat{z}}_d(\tau)) > 0$, then this trajectory will intersect the x-axis (i.e., $\dot{z}_a(\tau) = 0$) only once. The time instance of this intersection is denoted as t_s, and the corresponding z_a is $z_a(t_s)$. Now, there are three cases:

- If $z_a(t_s) \leq 0$, then it means that even the full deceleration is made, the quadrotor will still hit the platform. Thus $(z_a(t), \dot{z}_a(t))$ belongs to $\Omega_3(t)$, full deceleration has to be applied.
- If $z_a(t_s) > 0$, and $\dot{z}_a(t) \leq -\dot{z}_{rmax} - \dot{\hat{z}}_d(t)$, it means that with the full deceleration, the quadrotor will stop at a position higher than the platform. Thus, for time-optimality purpose, full deceleration does not have to be made at this point. However, the velocity of the quadrotor already reaches its maximum downward limit. Thus, $(z_a(t), \dot{z}_a(t))$ belongs to $\Omega_2(t)$, $\ddot{z}_a(t)$ is taken as $-\ddot{\hat{z}}_d(t)$ to maintain the landing velocity at its limit.
- If $z_a(t_s) > 0$, and $\dot{z}_a(t) > -\dot{z}_{rmax} - \dot{\hat{z}}_d(t)$, it means that with the full deceleration, the quadrotor will stop at a position higher than the platform. Furthermore, the downward velocity has not reached its limit yet. Thus, $(z_a(t), \dot{z}_a(t))$ belongs to $\Omega_1(t)$, $\ddot{z}_a(t)$ is taken as its lower limit to make a full downward acceleration.

The above decision process only requires the calculation of t_s, which is the solution of the following equation for τ

$$\dot{z}_a(t) + \ddot{z}_{rmax}(\tau - t) - \dot{\hat{z}}_d(\tau) + \dot{\hat{z}}_d(t) = 0. \quad (17)$$

As mentioned above, since the solution of the above problem is unique, any simple numerical method like bi-section method may be applied to obtain the solution.

V. Motion estimation module design

The previous two modules have shown that given sufficiently accurate estimations of the absolute height and velocity of the quadrotor as well as the platform motion, a fast and accurate landing maneuver can be achieved. In a typical VTOL UAV landing operation, the absolute height $z(t)$, velocity $\dot{z}(t)$ of the UAV and the oscillating platform motion are usually not directly measurable in real time. However, the relative distance between the UAV and the platform can be easily measured by computer vision-based approaches such as those in [9], [5]. Furthermore, the acceleration of the UAV can also be measured by an IMU (inertial measurement unit). Therefore, in this module, the estimates of $\hat{z}$, $\dot{\hat{z}}$, $\hat{z}_d$, $\dot{\hat{z}}_d$, $\ddot{\hat{z}}_d$ are to be obtained by only using the relative distance $z - z_d$ and the acceleration of the quadrotor $\ddot{z}$ as measurements based on the Unscented Kalman Filter similar to the work of [10]. The following assumptions are made:

- The platform motion is modeled as a single sinusoidal function, with $n = 1$ in (3).
- The amplitude A, frequency ω and phase ϕ are sufficiently slowly time varying (or time invariant).

Define the state vector to be estimated as $x = [x_1, x_2, x_3, x_4, x_5]^T$, where x_1 is the height of the quadrotor, x_2 is the velocity of the quadrotor, x_3 is the platform motion $A\sin(\omega t + \phi)$, x_4 is defined as $A\cos(\omega t + \phi)$, x_5 is the frequency ω. Let v be input, i.e., the acceleration of the quadrotor, which is different from the control input u in the previous sections. The output $y = x_1 - x_3$ is the relative distance between the quadrotor and the platform.

The system dynamics $\dot{x} = f(x, v)$ and output equation $y = h(x)$ can be written as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= v \\ \dot{x}_3 &= x_4 x_5 \\ \dot{x}_4 &= -x_3 x_5 \\ \dot{x}_5 &= 0, \\ y &= x_1 - x_3. \end{aligned} \quad (18)$$

Through a nonlinear observability analysis by checking the gradient of the Lie derivative of the dynamics equations, this system is locally observable. Given a close initial state, all the states can be estimated based on the standard UKF algorithm.

After obtaining the state estimations $\hat{x}(t) = [\hat{x}_1(t) \;\; \hat{x}_2(t) \;\; \hat{x}_3(t) \;\; \hat{x}_4(t) \;\; \hat{x}_5(t)]^t$, the absolute estimations of the position of the quadrotor $\hat{z}(t)$, the velocity of the quadrotor $\dot{\hat{z}}_d(t)$ and the platform motion $\hat{z}_d(t)$, velocity $\dot{\hat{z}}_d(t)$ and the acceleration $\ddot{\hat{z}}_d(t)$ can be estimated as (19).

$$\begin{aligned} \hat{z}(t) &= \hat{x}_1(t) \\ \dot{\hat{z}}(t) &= \hat{x}_2(t) \\ \hat{z}_d(t) &= \hat{x}_3(t) \\ \dot{\hat{z}}_d(t) &= \hat{x}_4(t)\hat{x}_5(t) \\ \ddot{\hat{z}}_d(t) &= -\hat{x}_3(t)\hat{x}_5^2(t) \end{aligned} \quad (19)$$

VI. Experimental results

A. Experiment setup

The quadrotor used in our experiment is the Nano plus quadrotor from KMel Robotics. The vertical motion of the

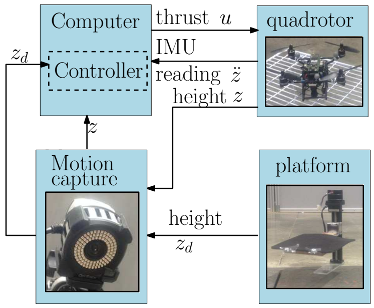

Fig. 2. This figure shows the experiment setup. The wave motion is generated by a linear stage, there are markers on the platform as well as the quadrotor. The motion capture system captures the markers feed the relative distance to the computer. The computer runs the control structure and sent the thrust command to the quadrotor wirelessly at 50 Hertz

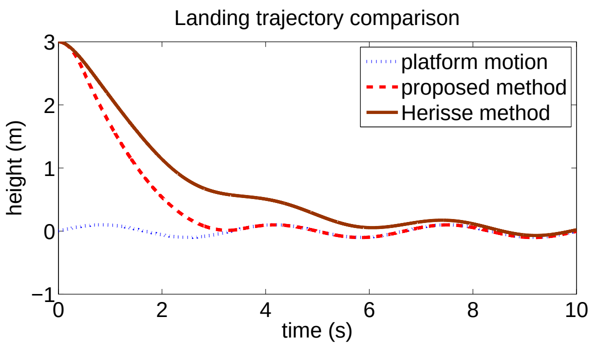

Fig. 3. This figure compares the height in the landing process. The proposed method (red line) decreases faster than [1] (brown line) method

platform is generated using an XSlide linear stage from Velmex Inc. For demonstration purposes, we measure the absolute heights of the platform and quadrotor using an external motion capture system called Optitrack. The Optitrack system sends the position information to a computer through the TCP connection. A computer runs the controller through Matlab and sent the control command wirelessly to the quadrotor through a ZigBee module. The experiment setup is illustrated in Fig.2.

B. Simulation comparison between the proposed method and previous method

This simulation demonstrates the trajectory comparison between proposed method with the method used in [1] for bench-marking purposes. It is key to note though that in [1], the only information available as feedback measurement is optical flow from the camera. The simulation environment is set to be the same as [1]. The platform motion is modeled as a sinusoidal signal as in (20), and the quadrotor starts landing at 3 meters above the platform.

$$z_d(t) = a\sin(2\pi f t) \text{ with } a = 0.1\text{m}, f = 0.3\text{s}^{-1} \qquad (20)$$

Fig. 3 shows the comparison between the trajectory z_r of proposed method and the method in [1]. From Fig.3 we note that the rate of convergence is faster for the proposed method because of the time optimal characteristic of the algorithm.

C. Experiment of ground effect compensation

This experiment demonstrates the effectiveness of the adaptation of the proposed controller. A comparison result of landing the quadrotor onto the ground with and without adaptation is presented. In this experiment, the tracking control and the trajectory generation module are activated. Since the ground is not moving, the relative distance measure is the same as the absolute distance. Thus, the motion estimation module is not needed. To show the effectiveness of the ground effect adaptation ability, we compare two algorithms: 1) the proposed method without ground effect adaptation ($\hat{\theta}$ is set as constant); 2) the proposed method with ground effect adaption . Figure. (4) shows a comparison of landing trajectory under adaptation, without adaptation and a reference trajectory. From the figure it's clear to note that landing with adaptation has a smaller tracking error when the quadrotor is approaching to the ground.

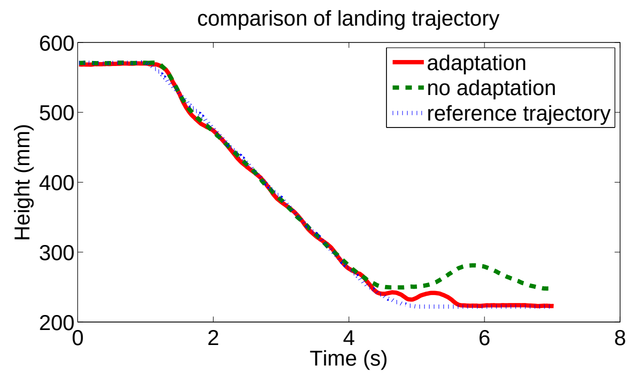

Fig. 4. This figure compares the landing trajectory under adaptation and without adaptation. When the quadrotor approaches to the ground without adaptation (green line), there is a huge deviation from the reference trajectory (blue line), which is caused by the ground effect. When the quadrotor lands with adaptation (red line), the error is much smaller

D. Landing on a sinusoidally oscillating platform

This experiment demonstrates the autonomous landing of the quadrotor onto a sinusoidally oscillating platform without the knowledge of motion parameters. The only measurements available is the relative distance $z(t) - z_d(t)$ and the acceleration of the quadrotor $\ddot{z}(t)$. In this experiment, all three modules described in this paper are activated. The landing process is divided into two phases. The first is the estimation phase, during which the motion estimation module estimates the absolute position, velocity and acceleration of the platform and the quadrotor. Fig. 5 shows the comparison of the estimated platform motion $\hat{z}_d(t)$ to the actual motion $z_d(t)$. From the figure it can be seen that the estimator has a good performance. After the motion estimation module gives a consistent estimation, which can be examined by testing whether the variation of the height estimation $\hat{z}(t)$ stays in a a predefined scope, then the landing process begins.

In the landing process, the trajectory generation module generates the reference trajectory $z_r(t), \dot{z}_r(t)$ and $\ddot{z}_r(t)$ at each iteration, then they are fed into the tracking control module as well as the height $\hat{z}(t)$ and velocity $\dot{\hat{z}}(t)$ estimation of

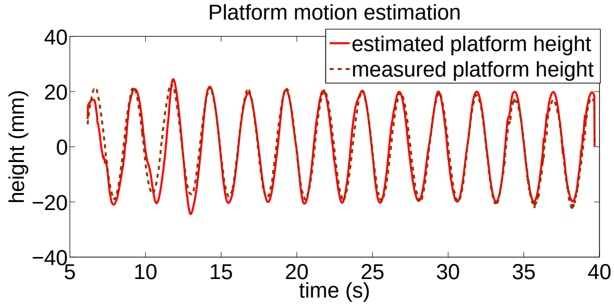

Fig. 5. This figure compares the estimation of the platform motion. The estimation (red line) converges to the real measurement (brown line).

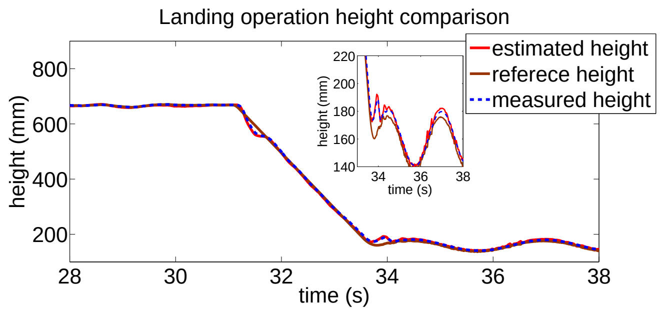

Fig. 6. This figure compares the actual landing trajectory (brown line) $z(t)$, the estimated trajectory (red line)$\hat{z}(t)$ and the reference trajectory (blue line) $z_r(t)$. It can be seen that in the landing process, the estimation still have a good accuracy and the actual trajectory follows the reference trajectory well. The landing operation finishes in less than 3 seconds

the quadrotor from the motion estimation module. Fig. 6 shows the reference trajectory $z_r(t)$, the estimated height $\hat{z}(t)$ and the actual height $z(t)$. From the result, it's clear that in the landing process, the quadrotor can track the reference trajectory well.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have presented a novel control structure that achieves the fast and accurate landing of VTOL UAV onto a vertically moving platform. The control structure consists of three modules: a motion estimation module; a trajectory generation module and a tracking control module. Comparative simulations and experiments show that the proposed control method can indeed achieve fast and accurate landing of quadrotor on a sinusoidally oscillating platform, and the performance is better than some of the previous methods.

In our future work, roll and pitch movements of the platform, which are more realistic in high sea state environments, will be considered. We will also extend the proposed algorithm to the case where the motion of the platform is a summation of multiple sinusoidal signals. Finally, the algorithm will be extended to 3D landing case where the initial position of the quadrotor differs from that of the platform in all the x, y and z directions.

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