Office Building Model Identification and Control Design

Matt Minakais¹, John T. Wen¹, Sandipan Mishra², Rongliang Zhou³, Zhikui Wang³, Amip Shah³

Abstract—This paper presents the identification of a lumped thermal model for one floor of a large office building consisting of partitioned cubicles and conference rooms. The space is instrumented with a large number of interior temperature sensors as well as supply-air temperature and flowrate sensors. Roof top solar radiation data is also available. The first step in the model identification process is to cluster the interior temperature measurements into zones. The zones form a thermal connectivity graph based on the physical location of the clusters. Each zone corresponds to a node in the graph with a corresponding thermal capacitance. The zones connect to adjacent zones and to the ambient via thermal resistances. Due to large windows on two walls and the ceiling, the model also needs to account for the radiant heat input into the space. We identify thermal capacitance, thermal resistance, and radiant heat transfer coefficients in the model by matching the predicted temperature trajectory with the measured data. As validation, the model shows reasonable prediction of temperatures on dates not used for identification. Using the identified model, we also consider the temperature control problem with real-world ambient temperature and solar data. By using our previously reported passivity-based temperature controller, we achieve tighter temperature regulation while consuming less energy as compared with the existing controller.

I. INTRODUCTION

In recent years, the rising cost of energy has led researchers to explore energy conservation in buildings, specifically in heating, ventilation, and air conditioning (HVAC) systems. A wide range of control strategies have been proposed to address this issue. Model predictive control has been particularly popular. Numerous papers, e.g., [1]–[3], are based on this approach and have demonstrated its effectiveness in simulation. However, these approaches also assume a priori model knowledge which may be difficult to obtain or may vary over time for building systems. In [4], [5], lumped building thermal models are used based on the analogy between heat transfer and current flow in an electrical circuit. Our work in [6], [7] uses a similar lumped thermal model represented as an undirected graph. By noting the inherent passivity of the system, we developed an adaptive control strategy which does not require explicit knowledge of the model parameters. Stability is guaranteed in this approach and the model information may be used to tune for performance in terms of transient response and energy efficiency.

In this paper, we apply a similar modeling approach to a real-world test facility. This facility consists of a large office building with numerous cubicles and several conference rooms. Our goal is to evaluate the applicability of the lumped thermal model to such a space. The lumped modeling approach does not capture mass transport within the building; however, our assumption is that the air flow is sufficiently slow such that the lumped model provides a sufficiently close approximation. To evaluate this assumption, we begin by breaking up the large space up into several zones with similar temperature trajectories. We then construct the thermal connectivity graph based on the physical adjacency between the zones and the ambient. Next, we use real-world data to find the best fit for the model parameters. To validate the model, we compare the model prediction and measured data for days not used for identification. By suitably constraining the model parameters, we are able to obtain reasonable prediction. Once the model is obtained, we demonstrate through simulation that the passivity-based adaptive control strategy provides tighter temperature control while consuming less energy, as compared to the performance of the existing HVAC system.

II. MODEL STRUCTURE

We consider a lumped heat transfer model using thermal resistance and capacitance as in [6]. This model represents heat flow as: \[ Q = \Delta T/R, \] where \( Q \) (in W) is the rate of heat transfer across the resistance \( R \) (K/W), and \( \Delta T \) represents the temperature difference (in K). In this model, thermal capacitance refers to a capacity of a space to store heat: \[ C \frac{d\Delta T}{dt} = Q, \] where \( C \) has the unit J/K. A single room/zone is modeled as a thermal capacitor, while a wall is modeled as a thermal resistor.

As in [6], we enumerate all thermal capacitors in the system as \( C_i \), and let \( T_i \) be the temperature of the \( i \)th capacitor, then the temperature dynamics governing \( T_i \) is given by

\[ C_i \frac{dT_i}{dt} = -\sum_{j \in N_i} R_{ij}^{-1}(T_i - T_j) + Q_i^{(e)} + Q_i^{(c)} \quad (1) \]

where \( N_i \) denotes all resistors connected to the \( i \)th capacitor, and \( R_{ij} \) is the thermal resistance between the capacitors \( i \) and \( j \). Additionally, we define the following matrices: \( Q_i^{(e)} \) is the external heat input due to ambient:

\[ Q_i^{(e)} = \begin{cases} -R_{i\infty}^{-1}(T_i - T_{\infty}) & \text{if } i \text{ adjacent to ambient} \\ 0 & \text{otherwise} \end{cases} \quad (2) \]
\( Q_i^{(c)} \) is the heat input due to control effort:

\[
Q_i^{(c)} = \begin{cases} 
  u_i & \text{if node } i \text{ is heated/cooled} \\
  0 & \text{otherwise}
\end{cases} \tag{3}
\]

We also define a matrix, \( D \), consisting of the values -1, 0, or 1, which describes the directionality of the graph structure. \( D \in \mathbb{R}^{n \times \ell} \), where \( n \) equals the number of nodes and \( \ell \) equals the number of links. Each column of \( D \) corresponds to a link in the graph, and the beginning and ending nodes are marked with 1 or -1, respectively. Thus, for each element in \( D \):

\[
D_{ij} = \begin{cases} 
  +1 & \text{if node } i \text{ begins link } j \\
  -1 & \text{if node } i \text{ ends link } j \\
  0 & \text{otherwise.}
\end{cases} \tag{4}
\]

By defining \( D \) in this way, we are able to re-write (1) as:

\[
CT = -DR^{-1}D^TT + B_0T_\infty + B(u + w) \tag{5}
\]

where \( C \in \mathbb{R}^{n \times n} \) is a diagonal, positive definite matrix consisting of the zone capacitances, \( R \in \mathbb{R}^{\ell \times \ell} \) is a diagonal, positive definite matrix consisting of the link thermal resistances, \( B_0 \in \mathbb{R}^n \) is a column vector with non-zero elements as the thermal conductance of nodes connected to the ambient, \( D \) is as the incidence matrix of the graph as described above, \( T_\infty \) is the ambient temperature, \( u \) and \( w \) are the controlled heat input and system disturbance, respectively, and \( B \in \mathbb{R}^{n \times n} \) is the corresponding input matrix. If there is no thermal storage between the zones (i.e., no walls), then \( B = I \), as every zone is actuated. For a connected graph, the open loop system (under constant ambient, heat input, and external disturbance) is exponentially stable.

### III. Test Facility

We consider an office building located in Palo Alto, California, measuring approximately 33\( \times \)76\( \times \)6 m and primarily containing cubicles and conference rooms. The general layout of the test bed can be seen in Figure 1. It is outfitted with the following sensors/actuators:

- **Ceiling temperature sensors:** There are more than 300 temperature sensors located on the ceiling, spanning the entire space.
- **Variable-Air-Volume (VAV) boxes:** There are 15 VAV’s equipped with sensors which report volumetric flowrate and supply air temperature of the HVAC unit.
- **Roof-mounted solar panels:** These panels provide solar energy data, which correlates to the amount of sunlight cast on the building.
- **Ambient temperature:** The local ambient temperature data is obtained from the local weather station.

All sensors report values on a 5-minute interval, with the exception of ambient data, which is reported every hour. We approximate 5-minute intervals of ambient temperature using linear interpolation. Reliable occupancy measurements are not available, so occupancy heat gain is not included in this model, though it is straightforward to incorporate this into the model’s disturbance term, \( w \), if data is available.

This particular space contains some architectural features which add a degree of difficulty to system identification. The northern and southern walls are comprised almost entirely of glass, while the eastern and western walls are standard. The ceiling also contains a significant number of glass windows. The large amount of glass on the exterior of the building adds the additional modeling challenge of radiant heat transfer. Additionally, the western wall is adjacent to an independently cooled building rather than ambient temperature. This, however, is accounted for by the model, since the thermal resistance for this wall (link) will be higher than other walls (i.e., heat transfer through this wall to/from ambient is low).

Figure 2 illustrates the significant contribution of radiant heat transfer to the system. This infrared thermal image was captured on August 15, 2013, while the ambient temperature was approximately 25°C.

![Fig. 1: Layout of test facility](image1.png)

![Fig. 2: Thermal image of windows along ceiling, illustrating significant radiant heat transfer (85.1°F = 29.5°C)](image2.png)

### IV. Faulty Sensor Data Removal

The measurements obtained from the ceiling-mounted temperature sensors are highly quantized (at 1°C increments) and some sensors give unreliable readings. The faulty sensors are generally of two forms; either a sensor reports a constant value for several days, or consistently reports unrealistic values. We classify a sensor as faulty if the readings remain essentially constant throughout the day or has sudden large jumps. Measurements in VAV, solar, and ambient data are generally reliable, and therefore not filtered. Figure 3 shows an example of data which has been sorted by this algorithm.

In this example, 71 sensors (out of 313) are eliminated. The 24-hour data sample shown is from August 14, 2012.
V. ZONE PARTITION

For a relatively large open space such as the one considered here, fluid dynamics simulation is usually used (e.g., as in data center cooling [8] and building simulation [9]). Such an approach is not amenable to control analysis and design. Instead, we investigate the feasibility of using a lumped modeling approach based on thermal resistance and capacitance. Our approach is to divide the space into zones, each represented as a thermal capacitor in the thermal network. These zones are connected by thermal resistance links, expected to have lower resistance values than wall resistances. To determine the zones and their boundaries, we apply the K-means clustering algorithm to the temperature data. The K-means method involves clustering a set of observation vectors \((x_1, x_2, \ldots, x_n)\) into \(k\) sets \((S = S_1, S_2, \ldots, S_k)\) as to minimize the variance within each cluster:

\[
\arg \min_k \sum_{i=1}^{k} \sum_{x_j \in S_i} \|x_j - \mu_i\|^2
\]

where \(\mu_i\) refers to the mean of \(S_i\). In our case, \(x_j\) is the temperature history of the \(j\)th sensor over a specified period, and \(\mu_i\) is the mean temperature in a set of sensors \(S_i\). K-means clustering is performed for \(k = 2, \ldots, 15\) clusters by using temperature data over 48 hours (August 2-3, 2012). A key criterion of an acceptable clustering is that the sensors within the cluster are spatially connected. The variance within the cluster also should be reasonably small. Based on these considerations, we choose five clusters, or \(k = 5\). Figure 4 shows the five-zone partition of the space, as well as the graph structure corresponding to the resulting 5 capacitive nodes and 12 resistive links.

![Fig. 4: Test facility - Zone layout (left) and corresponding zone graph structure (right)](image)

It is reassuring that the zones identified by k-means clustering correlate with the architectural design of the building. For example, zones 1 and 5 represent an area with a lower sub-ceiling and different types of vents. The physical boundary of this area aligns exactly with the lower boundaries of zones 1 and 5. Also, zone 2 is separated from zone 3 an 4 on the southern side by a large set of conference rooms and a walkway. Additionally, a large open area and staircase separate the southern half of the building, which correlate precisely to the zone boundary between zones 3 and 4. With the graph structure defined as in Figure 4, and the directionality of each link between two nodes defined as pointing from the node of lower index to the node of higher index, the graph incidence matrix \(D\) is:

\[
D = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

The temperature of each zone, \(T_i\), is defined as the average of all temperature readings within that zone (excluding faulty sensor measurements that have been eliminated). Figure 5 shows the band of temperature measurements for each zone, as well as each zone’s average temperature, \(T_i\). The 48-hour data sample shown was recorded on August 8-9, 2012.

![Fig. 5: Temperature sensor measurements in each of the five zones and their average values over a 48-hour period](image)

VI. SYSTEM IDENTIFICATION

Our first attempt is to consider the model (5) of the form:

\[
CT = -DR^{-1}D^TT + B_0T_\infty + B(u + K_aT_\infty)
\]

where the disturbance term, \(w\), is replaced by a radiant heat term, \(K_aT_\infty\). The purpose of this term is to account for
the radiant heat added to the building via direct sunlight. We originally use $T_\infty$ as a rough estimate of sunlight, the justification being that there will generally be more sunlight during daytime hours, when the ambient temperature is highest. $K_a$ serves as a model-specific proportionality coefficient which can be identified in the same fashion as $C$ and $R$.

The use of $T_\infty$ to estimate sunlight proved to be un-dependable for two reasons. Firstly, $T_\infty$ contains non-zero values at night, when there is indeed no sunlight and thus no radiant heat contribution. Secondly, $T_\infty$ does not seem to correlate with sunlight due to the presence of clouds (i.e., cloud cover causes the amount of sunlight to change much more dramatically than the ambient temperature). To correct this, the $T_\infty$ term was replaced with $P_s$, which is defined as the power generated from the roof-mounted solar panels:

$$CT = -DR^{-1}DT + B_0T_\infty + B(u + K_aP_s)$$  \hspace{1cm} (9)

Figure 6 shows an example of the recorded data from a roof-mounted solar panel. As expected, the power generated is zero when the sun is down and reaches its maximum value around 1:00 PM.

First, we perform system identification using $T_\infty$ to estimate sunlight as in (8). The result based on Aug 2-3 data is shown in Figure 7. This approach provides a good match for the same days used for identification, however, the model fails when verified on different days (August 14-15, 2012) as shown in Figure 8. This confirms that radiative heat transfer cannot be accurately predicted by $T_\infty$.

For parameter identification, we use the finite difference approximation of (9):

$$T(k+1) = T(k) + C^{-1}T_s[-DR^{-1}DT(k) + B_0T_\infty + B(u + K_aP_s(k))]$$  \hspace{1cm} (10)

where the sampling interval $T_s$ is set to 5 minutes to match with the sampling rate of the temperature sensors. The error, $e$, is defined as the Euclidean norm of the difference between measured and predicted (using (10)) temperatures in each zone over the 48-hour period (Aug 2-3, 2012). We use a standard interior point nonlinear programming solver to find the $2n + \ell$ values of $(R, C, K_a)$ which minimize $e$. To ensure realistic values, we constrain the model parameter values to the following ranges:

$$10^{-6} \leq R \leq 10^{-1}$$

$$10^5 \leq C \leq 10^{10}$$

$$10^{-2} \leq K_a \leq 10^3$$  \hspace{1cm} (11)

Next, we identify a new set of model parameters using solar panel data, $P_s$, to predict radiative heat transfer. Figure 9 demonstrates the success of this method by comparing modeled data from August 2-3, 2012 with the measured data from those days. For validation, the same model identified with August 2-3 data is applied to the measured data from August 14-15, 2012. In this case, the model prediction matches well with the actual measurements. These results are shown in Figure 10.

**VII. Control Simulation**

As an assessment of the existing HVAC controller, we apply the adaptive control strategy from [7] to the identified model (9) using real-world values for the ambient temperature $T_\infty$ and solar panel data $P_s$. We set the desired room temperature at a constant 22°C and assume a constant supply-air temperature, $T_s$, of 18°C. The solar panel data and ambient temperature profile used for simulation can be seen in Figure 11.
The adaptive feedforward heat input, \( \hat{u}^* \), is given by:
\[
\hat{u}^* = \hat{F}_0 y_{\text{des},i} + \hat{F}_1 T_\infty + \hat{F}_2 P_s
\]  
(12)
where \( \hat{F}_0, \hat{F}_1, \) and \( \hat{F}_2 \) are gains adaptively updated by:
\[
\dot{\hat{F}}_0 = -\Gamma_0 (y - y_{\text{des},i}) y_{\text{des},i}^T
\]  
(13)
\[
\dot{\hat{F}}_1 = -\Gamma_1 (y - y_{\text{des},i}) T_\infty
\]  
(14)
\[
\dot{\hat{F}}_2 = -\Gamma_2 (y - y_{\text{des},i}) P_s
\]  
(15)
and \( \Gamma_0, \Gamma_1, \) and \( \Gamma_2 \) are positive definite gain matrices. Note that we cannot directly control the heat input \( u \); rather, in practice we control the volumetric flowrate delivered by each VAV. Given a desired heat input, the corresponding flowrate for each zone, \( \hat{m}_{s,i} \), is calculated by:
\[
\hat{m}_{s,i} = -K (y_i - y_{\text{des},i}) + \frac{\hat{u}_i^*}{c_p (T_s - T_i)}
\]  
(16)
Figure 12 shows the system behavior of the test facility (using the existing controller) for 48 hours beginning on August 14, 2012 at 12:00 AM. Note that there are large temperature variations throughout the 48-hour period, as well as large differences between zone temperatures. The controlled heat input is shown in Figure 13. Notice that energy is being wasted by the existing controller due to large overshoot, as well as heating one zone while the others are cooled. In the test facility there are VAV ducts in zones 1 and 5 for which data is not available, which explains the seemingly low control effort in these zones. This issue does not seem to significantly effect the system identification, so we proceed with the assumption that these zones are simply under-powered compared to other zones.
The heat input for the adaptive controller case is shown in Figure 15. Note the aggressive cooling during the daytime with high solar heat gain and ambient temperature (see Figure 11), which avoids the zone temperature overshoot as seen under the existing controller in Figure 12. For the adaptive controller simulation, constraints are placed on \( \dot{m}_s \) to match the real-world system. A consequence of these constraints is larger temperature setpoint deviation for zone 1 as seen in Figure 14. This is due to the fact that zone 1 has the fewest amount of vents, leading to a lower saturation level of the control input in this zone. At about 6:00 PM on both days, we observe a slight overshoot in zones 2-5, caused by the rapid decrease in ambient temperature and radiant heat as shown in Figure 11.

![Fig. 14: Simulated zone temperatures using environmental data collected on August 14-15, 2012 and adaptive controller](image)

![Fig. 15: Simulated controlled heat input using environmental data collected on August 14-15, 2012 and adaptive controller](image)

**VIII. Conclusion**

This paper demonstrates successful modeling of a large office building using a lumped heat transfer model. Though the space is relatively open, we show that a thermal model consisting of interconnected thermal resistance and capacitance adequately captures the temperature evolution of the building with controlled heat input, ambient heat transfer, and solar heat gain. The model parameters, consisting of thermal resistance, capacitance, and solar heat gain coefficients, are identified through constrained minimization using measured data. The model is also validated on days not used in identification. We apply our passivity based adaptive controller to the identified model and show that it is feasible to achieve tight temperature control while consuming less energy than the existing HVAC system.

We are currently investigating the adaptation of the desired zone temperature \( y_{\text{des}} \) to balance between energy consumption and human comfort. This will involve combining individual user feedback signals (e.g., comfortable, too hot, or too cold) with energy minimization to determine the zone temperature setpoints.

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**REFERENCES**


