A Comfort Zone Set-Based Approach for Coupled Temperature and Humidity Control in Buildings

Charles C. Okaeme¹, Sandipan Mishra¹, John T. Wen²

Abstract—This paper presents a modeling and control strategy for comfort zone set-based control of temperature and humidity in buildings. We first propose a coupled model for humidity and temperature dynamics based on lumped parameter analysis. The interconnection of rooms/zones is captured through an undirected graph, with rooms represented as capacitances and walls and doors/windows as resistances. Unlike traditional RC-models, however, this model captures both mass and heat transfer between zones as well as the bilinearity in the input mass flow-rate. Key parameters are identified by the model, such as mass (and thermal) conductance between zones as well as mass (and thermal) capacitance and this model structure is then validated using physics-based Computational Fluid Dynamics (CFD) simulations. The control inputs to the system are the mass flow rates into each zone and the control objective is to drive the system state into a comfort zone set (a humidity and temperature region defined on the psychometric chart). The dynamic system is shown to be passive, hence any passive controller is stabilizing and able to drive both temperature and humidity to steady states within the thermal comfort region for given ambient conditions. We then propose a set-based (passive) controller to regulate the system outputs within the comfort region. Simulation results from implementing the controller on the lumped model are then compared with CFD simulations, for a design model of an existing experimental 6-room test bed. The proposed controller design methodology is also shown to be model-independent with results of the CFD simulations verifying this feature.

I. INTRODUCTION

Providing a comfortable indoor environment for building occupants is the primary goal of Heating, Ventilating and Air-conditioning (HVAC) systems. The system performance has significant influence on inhabitant’s health, productivity and satisfaction [1]. Intelligent building control systems are thus designed to efficiently regulate variables in the building that affect human comfort based on feedback from sensor measurements. The main indoor comfort parameters categories include thermal, air quality and illumination levels. Typically, the most significant thermal comfort parameters to be controlled are temperature and relative humidity [2].

There has been substantial research interest in modeling and control design for indoor environment regulation. Most of these works have focused on temperature control alone when modeling energy transfer in buildings. A recent review performed by Shaikh et al [3] of state-of-the-art building comfort control systems revealed that comparatively few works consider relative humidity in modeling and control for indoor comfort. Apart from relative humidity being a key parameter in the human comfort index, the strong coupling of temperature and humidity dynamics during thermal exchange also needs to be adequately addressed for ensuring thermal comfort and energy optimization in control implementation.

Naturally, it is not always possible to drive the indoor environment to any desired temperature and humidity level using only temperature-based control schemes. To address this, some recent work has explored independent control of temperature and humidity [4], while others have considered temperature, humidity and other metrics as part of the overall comfort management problem [5]. Most of the recent humidity control approaches reported in the literature employ fuzzy reasoning and Model Predictive Control (MPC) strategies.

In many typical building systems, however, only mass-flow rate into a room is controlled, without direct control of both temperature and humidity. Therefore, this work focuses on regulation of both temperature and humidity by controlling supply air mass flow rate alone. The control strategy employed here is to ensure thermal comfort by maintaining temperature and humidity within a set that represents the comfort zone as recommended by ASHRAE [6]. We model the thermal comfort problem as a coupled temperature and humidity dynamical system network with an input-state bilinearity. The network of interconnected zones within a building is described using electrical circuit analogy [7]. This enables us to apply passivity-based control schemes for this problem, which is advantageous for large scale interconnected systems with significant parametric uncertainties. The moisture and heat transfer model presented here is a lumped capacitance model, allowing application of graph theory in simplifying the network control problem. Though lumped capacitance models are relatively simplistic for energy transfer models, the control architecture in this paper is not model parameter-dependent and this feature is highlighted by applying the controller in a CFD simulation. The proposed dynamic model captures essential coupling of humidity and temperature and may be utilized by other control and optimization techniques, such as MPC. The controller performance is illustrated using both lumped model and CFD simulations of a multi-zone building (supplementary material provides summary of this work with video captures of some implementation results).

The paper is organized as follows: Section II presents the problem description. In Section III we develop the system model and describe the passivity structure. Sections IV and V discuss controller design and stability analysis, Section VI illustrates our key results and Section VII is the conclusion.

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II. Problem Description

Figure 1 illustrates a typical coupled moisture-temperature control problem in a building. The system represents a zone or network of zones in a building where the humidity ratio $w$ and temperature $T$ are to be driven and maintained at a desired equilibrium (or a set of comfortable equilibria) using only one control input - the mass flow rate $\dot{m}$ of the supply air (maintained at $T_s$ and $w_s$). Exogenous inputs to the system include the ambient conditions ($w_\infty$, $T_\infty$) and the heat/moisture loads ($\dot{Q}_l$, $\dot{W}_l$) which may be from individuals, sunlight, etc. The objective is to design the controller (in a relatively model-independent fashion) such that closed loop stability is guaranteed and that the system output reaches and stays within the desired set of comfortable conditions.

III. System Model

A. System Inter-connectivity & Graph Structure

Although the proposed methodology is generalized to an arbitrary interconnection of rooms, we will use a 6-room building test bed [8] as a basis for illustrating the model and simulation examples presented in this paper. Figure 2 shows the building’s layout with inter-connectivity of thermal and mass exchange. For the connected graph model, each room is typically modeled as a single thermal/mass capacitance, while all other connections (i.e. through walls) are modeled as 3RC networks. A slightly simpler network is built by absorbing the adjacent wall capacitances into the room thermal capacitances and the resulting graph is also shown in Fig. 2. Nodes 1-6 correspond to the numbered rooms, while the links correspond to thermal and mass resistances from walls and doors/windows respectively. Links having no end-node represent exterior walls (for thermal exchange) or windows open to the ambient (for thermal/mass exchange).

B. Single Room Coupled Temperature and Humidity Model

We begin with modeling a single room and consider a lumped parameter model for thermal and mass dynamics in the room space (Fig. 3). First of all consider a zone with moisture and heat supply, with an open door/window i.e. the only disturbances are ambient temperature and moisture. The following assumptions are made: (1) No accumulation of mass in the room. (2) Fluid thermo-physical properties in the room and at the outlet are equal i.e. well-mixed system. (3) Radiation heat transfer between walls, windows and/or other objects in the room is negligible. However, this can be added as a disturbance without affecting the model structure. (4) Thermal contribution of moisture is approximated by latent heat of vaporization. This is useful for addressing the non-linearity of the energy equation [9]. Applying mass and energy conservation laws, the dynamic model for a single zone is of the following form:

$$M \dot{w}_r = \dot{m} (w_s - w_r) - G (w_r - w_\infty) + W_{gen} \quad (1)$$

$$C \dot{T}_r + \beta M \dot{w}_r = \dot{m} C_p (T_s - T_r) + \dot{m} \beta (w_s - w_r) - K (T_r - T_\infty) + Q_{gen} \quad (2)$$

where $C$ is the thermal capacitance of the room, $M$ is mass of air in the room and represents a mass capacitance. $w_s,$
$T_s$ are supply moisture content and temperature respectively while $w_s$, $T_s$ are corresponding room conditions (moisture content/humidity ratio is the ratio of mass of water vapor to mass of dry air). $W_{gen}$ and $Q_{gen}$ are moisture and heat generation within the room. $C_p$ is specific heat of air at constant pressure. $\beta$ is latent heat of vaporization of water. $K$ is wall thermal conductance. Similarly, we define the parameter $G$ as mass transfer conductance between the room and ambient. Equation (1) describes the overall moisture rate of change to be a contribution of moisture supply to the room (first term on right hand side), moisture generation within the room (third term) and moisture loss to ambient through open doors/windows (second term). Equation (2) explains that the increase in sensible heat (first term on left hand side) and latent heat (second term on left hand side) within the room is determined by the sensible and latent heat supply (first and second term on right hand side respectively), heat generation within the room (fourth term on right hand side) and heat loss to the ambient through walls and doors/windows (third term on right hand side). We may rewrite the energy conservation law, Equation (2), to represent the temperature evolution of the air in the room by substituting in Equation (1) on the left hand side. With this, the dynamic model of the single room (dropping the 'r' subscript) becomes

$$
M\dot{w} = \dot{m}(w_s - w) - G(w - w_m) + \dot{W}_{gen}
$$

$$C_T\dot{T} = \dot{m}C_p(T_s - T) + \dot{\beta}G(w - w_m) - KT(\beta T - T_m) + \dot{Q}_{gen} + \dot{\beta}W_{gen}
$$

### C. Passivity Analysis

**Definition:** A system with state $x$, input $v$ and output $y$ is called passive [10] if there exists a continuously differentiable storage function $V(x) > 0$ such that $v^T y \geq V$. 

**Proposition 1:** The system described by Equations (3) and (4) with state $x$, input $v$ and output $y$ defined as

$$x = \begin{bmatrix} w \\ T \end{bmatrix}, v = \begin{bmatrix} v_m \\ v_e \end{bmatrix}, y = \begin{bmatrix} \alpha w \\ T \end{bmatrix}
$$

with no moisture/heat generation within the room, is strictly passive from the input $v$ to output $y$ for $\alpha > \frac{1}{2}B^2G$, where $\alpha$ is a scaling factor for the output. 

**Proof:** We study the passivity of the system with respect to the equilibrium point $x_0 = [w_m \ T_m]^T$. Define the deviation variables $w' = w - w_m$ and $T' = T - T_m$. Define a rescaled output $y^* = [\alpha w' \ T']^T$ and an energy storage function:

$$V = \alpha \frac{1}{2} Mw'^2 + \frac{1}{2}C_T T'^2
$$

$$\dot{V} = \alpha w' (v_m - Gw') + T' (v_e - KT' + \beta Gw')
$$

Now $Z(t) = \begin{bmatrix} w' & T' \end{bmatrix} \begin{bmatrix} \alpha G & 0 \\ -\beta G & K \end{bmatrix} \begin{bmatrix} w' \\ T' \end{bmatrix}$. 

Since $\alpha > \frac{1}{2}B^2G$, $Z(t) > 0 \forall t$ and thus the system is strictly passive with respect to the equilibrium. 

**Remark:** The concept of passivity is motivated by physical systems that store or dissipate energy, such as passive electrical networks and mass-spring-damper type structures, where $V(x)$ represents an energy function. The Passivity Theorem states that the negative feedback connection of two passive systems $H_1$ and $H_2$ with positive definite and radially unbounded storage functions $V_1(x)$ and $V_2(x)$ respectively, is passive, and the equilibrium of the interconnection is stable in the sense of Lyapunov. This will be later exploited for the design of a model-independent stabilizing controller.

### D. Extension to Multiple Zone Models

For a system of multiple zones, with no moisture or heat generation within each zone, we can rewrite the conservation laws as follows:

$$M_i\dot{w}_i = -\sum_{j \in \mathcal{N}_i} G_{ij}(w_i - w_j) + n_i^{(e)} + n_i^{(c)}
$$

$$C_i\dot{T}_i = -\sum_{j \in \mathcal{N}_i} K_{ij}(T_i - T_j) + \dot{Q}_i^{(e)} + \dot{Q}_i^{(c)} + \beta \left( \sum_{j \in \mathcal{N}_i} G_{ij}(w_i - w_j) - n_i^{(e)} \right)
$$

where $\mathcal{N}_i$, $\mathcal{N}_j$ denote neighboring nodes (zones) to the $i$th node (zone) for mass and thermal exchange respectively, $n_i^{(e)}$ is the external moisture input:

$$n_i^{(e)} = \begin{cases} -G_{i0}(w_i - w_m) & \text{if open to ambient;} \\
0 & \text{otherwise.} \end{cases}
$$

and $n_i^{(c)}$ is the control moisture input:

$$n_i^{(c)} = \begin{cases} v_{m,i} & \text{if node (zone) is humidified/dehumidified;} \\
0 & \text{otherwise.} \end{cases}
$$

Similarly, $\dot{Q}_i^{(e)}$ and $\dot{Q}_i^{(c)}$ are external and control heat input respectively. Using the graph, the overall system model is

$$\begin{align}
\dot{M}\ddot{w} &= -LGL^T W + B_{m0}w_m + B_m v_m \\
\dot{C}T &= -DKD^T T + B_{c0}T_m + B_v v_e + \beta (LGL^T W - B_{m0}w_m)
\end{align}
$$

where $M$ and $C$ are diagonal, positive definite matrices consisting of the moisture and thermal capacitances, $G$ and $K$ are diagonal, positive definite matrices consisting of the link mass and thermal conductances. $L$ is the incidence matrix for moisture exchange (Fig.2) whose values are determined by the graph structure:

$$L_{ij} = \begin{cases} +1 & \text{if } j \in \mathcal{L}_i^+ \\
-1 & \text{if } j \in \mathcal{L}_i^- \\
0 & \text{otherwise.} \end{cases}$$
where $\mathcal{L}_i^+$ ($\mathcal{L}_i^-$) denotes the set of links for which node $i$ is the positive (negative) end. For the example in Fig. 2, all arrowheads (tails) correspond to $\mathcal{L}_i^+$ ($\mathcal{L}_i^-$). $D$ is the corresponding incidence matrix for thermal exchange, $B_{m0}, B_{e0}$ are column vectors with non-zero elements as the mass and thermal conductance of nodes connected to the ambient, $v_{m,i}$ and $v_{e,i}$ are moisture and heat input to each zone respectively, and $B_{m}, B_{e}$ are the corresponding input matrices. Note that since $L$ and $D$ are full row rank, $LGL^T$ and $DLD^T$ are positive definite [11]. For this multi-zone system, our interest is in moisture and temperature regulation of zones directly affected by active heating/cooling and humidification/dehumidification. Therefore, the system state $X = [W \ T]^T$, input $v = [v_m \ v_e]^T$ and output $Y = [\alpha B_m^T W \ B_e^T T]^T$ consisting of stacked up humidity ratio and temperature elements.

Remark: It may be noted that a similar proof for passivity of the interconnected system can be developed for the multiple zone model, but is omitted here for brevity.

IV. TEMPERATURE SET POINT CONTROL

An approximate comfort region range for temperature and humidity ratio is depicted in Fig. 2 [ASHRAE [6]]. Figure 2 shows examples of state trajectories for a temperature-only feedback controller (see inset) with different set points, where the moisture input is allowed to run freely. Through this example, it is clear that although the desired temperature is reached in all three cases, the final temperature and humidity conditions do not fall within the comfort zone. Thus using temperature-only feedback may result in undesirable indoor conditions.

With the controller in this example, driving the system into the comfort region will be largely dependent upon favorable ambient humidity conditions or having another independent moisture control source. Thus, we consider a more efficient method in the following section that uses both temperature and humidity measurement in the feedback control algorithm.

V. COMFORT ZONE SET-BASED CONTROL

Our goal is to drive the temperature and humidity towards a set that captures the comfortable zone. This is to be achieved using only mass flow rate as the control input. We first define the zone as an ellipse of adequate size to capture the zone (Fig. 3). This is a reasonable approximation since any state inside the ellipse will be within the zone. We propose a feedback controller of the form

$$m = \begin{cases} K_p (X_c - X) & \text{if } d > 0; \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

where $K_p = [\bar{w} \ k_T]$ is the proportional gain and $X_c = [w_c \ T_c]^T$ is the center of the ellipse $E$. $d$ is the distance of the state from the ellipse boundary computed as:

$$d(X,E) = (X_c - X)^T Q (X_c - X) - 1 \quad (17)$$

where $Q = RQ'R^T$ is the rotation of the ellipse to lie adjacent to the zone boundary inclination. $R$ is the rotation matrix and $Q'$ is the matrix of the major and minor axes of the ellipse. To drive the state well into the comfort zone before shutting off supply, the distance condition may be implemented with an inner ellipse as a hysteresis bound. Simulation examples of the controller are shown in later sections.

![Approximate temperature and humidity comfort zone with enclosed ellipse set for control law](image)

Fig. 5. Approximate temperature and humidity comfort zone with enclosed ellipse set for control law.

A. Closed Loop Stability Analysis

Previously we showed that the mapping from $[v_m \ v_e]^T$ to $[\alpha w \ T]^T$ is passive for $\alpha$ large enough. However, the actual input for most HVAC systems is not $[v_m \ v_e]^T$, instead it is the mass flow rate, $m$, which is related to $[v_m \ v_e]^T$ through

$$\theta = [\theta_1 \ \theta_2]^T = [(w_s - w) \ C_p (T_s - T)]^T \quad (16)$$

where $w_s$ and $T_s$ are the supply moisture and temperature, respectively. Thus, $\theta(t)$ connects $m$ to the synthetic inputs $[v_m \ v_e]^T$, $\theta(t)$ is time-varying but sign-definite. Furthermore, it is not dependent on the model parameters (only on the specific heat capacity of air. $C_p$ and the supply air temperature/moisture ratio and current conditions in the room).

Considering Fig. 6, we can see that the closed loop system will be stable if the mapping from the output $[\alpha w \ T]^T$ to $[v_m \ v_e]^T$ is dissipative, based on the passivity theorem. Note that this mapping includes the controller $\tilde{K}_p$ and $\theta$, see Fig. 6. Thus, the controller ($\tilde{K}_p$) must be designed so as to ensure passivity of the cascaded blocks $\theta \tilde{K}_p$, as shown in the figure.

Proposition 2: The mapping from the output $y$ to the synthetic input $v$ through the controller $\tilde{K}_p$ is stabilizing if

$$\frac{K_p}{k_T} = \alpha c(t),$$

where $c(t) = \theta$. 

![Temperature feedback control with open loop moisture supply at various set points. Examples show that set point is reached, but overall temperature and humidity conditions do not reach thermal comfort zone.](image)

Fig. 4. Temperature feedback control with open loop moisture supply at various set points. Examples show that set point is reached, but overall temperature and humidity conditions do not reach thermal comfort zone.
**Fig. 6.** Stability of the closed loop. The mapping from the output \([\alpha w \ T]^{T}\) to the synthetic input \([v_m \ v_e]^{T}\) through the controller is passive with \(\theta \bar{K}_p\) positive semi-definite.

**Proof:** Consider the equivalent feedback system shown in Fig. 6. Denote \(\frac{k_w}{\alpha}\) as \(\bar{k}_w\) and \(\bar{K}_p = [\bar{k}_w \ k_T]\). Then, the map from \([\alpha w \ T]^{T}\) to \([v_m \ v_e]^{T}\) is described by:

\[
\begin{bmatrix}
    v_m \\
    v_e
\end{bmatrix}
= \theta \bar{K}_p
\begin{bmatrix}
    \alpha \left( w_e - w \right) \\
    T_e - T
\end{bmatrix}
\]  

(18)

With the controller \(\bar{K}_p\) chosen as described above, the term \(\theta \bar{K}_p\) in Equation (18) becomes \(\theta q \theta^{T} \succeq 0 \forall t\), where \(q\) represents a scaling for the gains (since \(\bar{k}_w/k_T = \bar{\theta}_1/\bar{\theta}_2\) or \(\bar{K}_p = q\theta^{T}\)). \(\theta \bar{K}_p\) is positive semi-definite, hence the mapping is passive and the closed loop system is stable. ■

**Proposition 2** suggests that we can choose time-varying proportional gains to satisfy this relationship at every instant to guarantee passivity and ensure stability of the system. However, if \(c(t)\) does not vary significantly then constant gains can be used without affecting the passivity structure.

**Remark:** This result shows that we do not need to rely on the model parameter accuracy for guaranteeing stability of the closed loop system. Moreover, there is a large family of stabilizing controllers to choose from for control of indoor environment conditions in buildings. Any available model information may be used to design the controller towards an optimization objective, such as energy efficiency, without compromising the passivity configuration.

**VI. EXAMPLE**

**A. Lumped Capacitance Model Simulation**

We use the six-room building example with the corresponding thermal/mass network graph in Fig. 2 to illustrate the controller performance. As earlier mentioned, wall and room thermal capacitances are combined to give a lumped thermal capacitance for each zone. Consequently, thermal resistance between any two connected zones is a lumped conductive/convective thermal resistance. For the following examples, thermal and mass conductances \((K_{ij} \text{ and } G_{ij})\) between interconnecting rooms and between each room and the ambient are determined from system identification tests on our CFD model of the test bed. For the 6 rooms, there are 6 thermal capacitive and 6 mass capacitive elements, 14 thermal resistance elements and 10 mass resistance elements. Hence, the thermal incidence matrix \(D\) has a dimension of \(6 \times 14\) and the mass incidence matrix \(L\) is \(6 \times 10\). Building properties, such as room dimensions and wall thermal properties used in calculations are obtained from [8]. Different scenarios for ambient temperature and humidity are obtained from weather history reports of various cities [12].

**Fig. 7.** Single room: Control performance with fixed/varying gain \(K_p\) \((\alpha = 10^3)\) and variation of \(\alpha c(t)\) with time for all settings \((k_w/k_T = \alpha c(t))\). "Set-based" means ellipse distance condition is imposed (with hysteresis). Ambient conditions of average weather in Phoenix, Arizona for November 12, 2015. Relative humidity at 20%, initial states \(T_0 = 292.15 \text{ K}, w_0 = 0.0025\). No saturation on mass flow input.

Figure 7 shows a transient profile of temperature and moisture content in a single room case with fixed and varying gain controllers. Note that the quantity \(\alpha c(t)\) does not depend on the model, so it can be obtained whether a varying gain controller is used or not (the model parameter-independence of the controller is unaffected). In Fig. 7 it is seen that \(\alpha c(t)\) does not vary significantly over the test duration. Hence, the controller performance is similar for both \(K_p\) and \(K_p(t)\).

**B. CFD Simulation**

Using ANSYS 16\textsuperscript{®}, a CFD analysis is performed to compare with the lumped model simulation. The CFD model uses the same controller as the lumped model. The 6-room building is drawn and meshed in a three dimensional domain and suitable boundary conditions with material specifications are imposed to match the physical test bed.

**C. Comparison of lumped capacitance simulation with CFD**

The model parameter-independent feature of the controller is evident from CFD simulation results shown in Fig. 8 where it is seen that state trajectories for the five controlled rooms enter the comfort zone. Comparing graphs of lumped model and CFD simulation results, the lumped model assumptions (combined with lumped room and wall capacitances) contribute to the inaccuracy of the model. However, agreement between both results is good considering the overall behavior of humidity ratio and temperature profiles.

**VII. CONCLUSION**

This work presented a model parameter-independent set-based controller for regulating temperature and humidity to
achieve indoor thermal comfort. The controller uses mass flow rate of air as the only input for feedback control, with temperature and moisture content as outputs. It has been shown that regulating temperature alone is not sufficient to maintain building indoor conditions in the comfort zone. The closed loop system with the proposed controller is shown to be stable and the graph network adequately captures the interaction between multiple zones in the building. Since the building moisture and thermal control problem is inherently passive, many different stabilizing controllers may be utilized, with additional objectives such as energy minimization and incorporating the dynamics of other HVAC system elements. Future work includes energy minimization considerations for the controller design.

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